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Title: Sensitivity Analysis using Continuous Sensitivity Equation Method (CSE)

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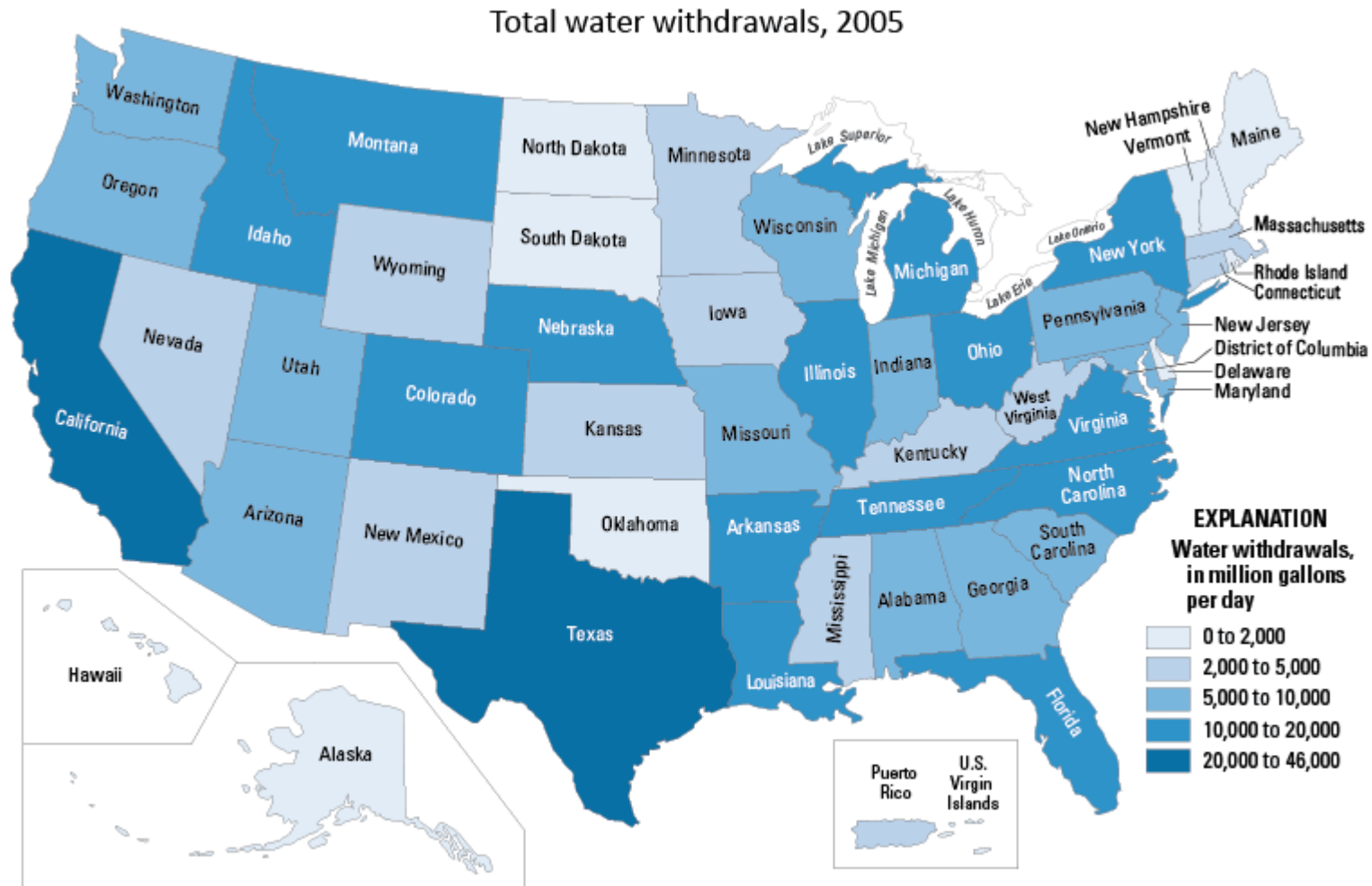
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# Sensitivity Analysis using Continuous Sensitivity Equation Method (CSE)

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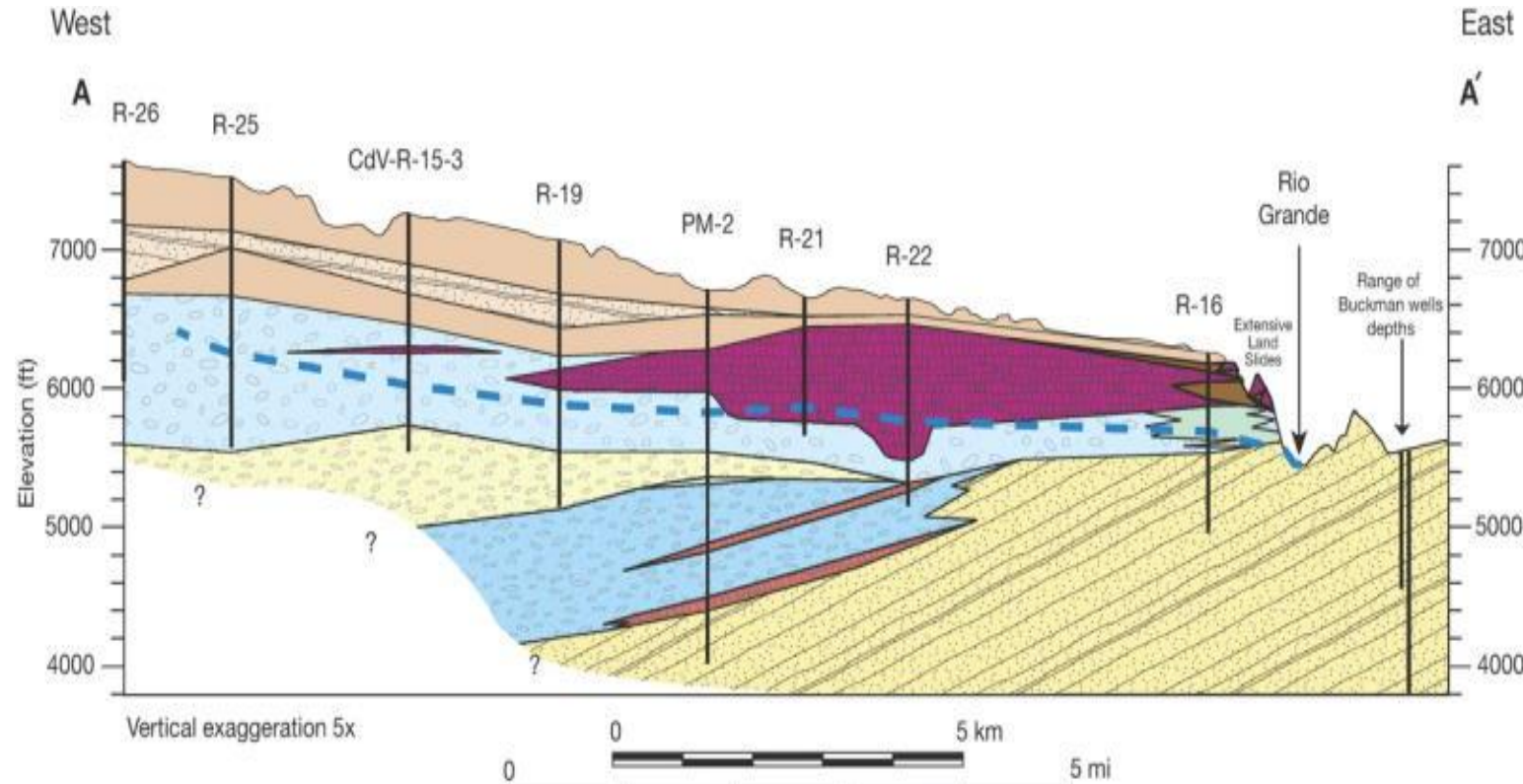
# Groundwater Models are Great



Source: USGS

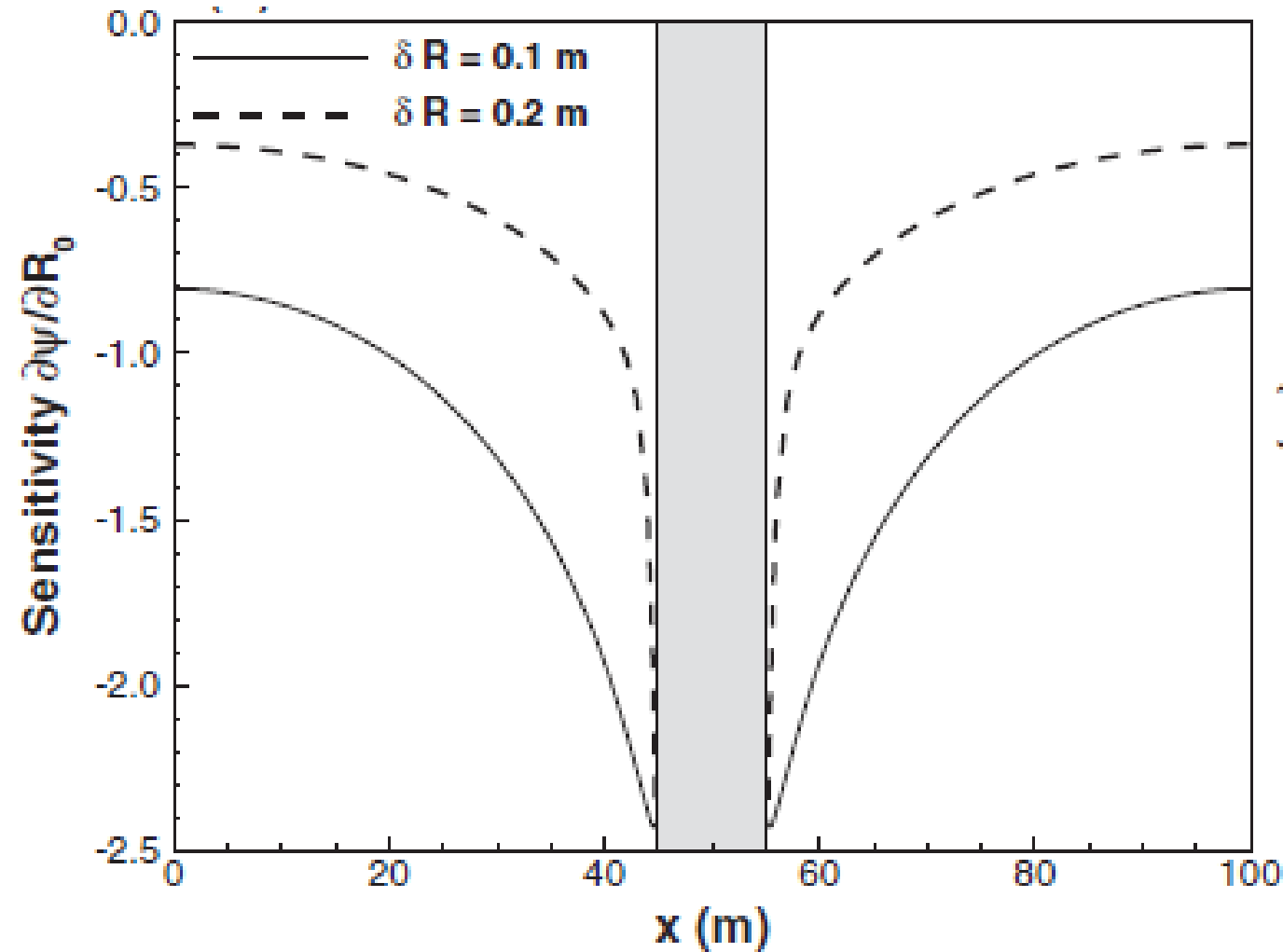
# ...But

- Boreholes are miles apart
- How sensitive are flow and transport are to these boundary locations?



# One Solution: Finite Difference Method

- Different nodal locations
- Hard to determine best perturbation



# Better Solution: CSE

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- Continuous Sensitivity Equation
- Can make direct nodal comparisons
- Not dependent on perturbation
- Get more accurate sensitivities

# How CSE Works

- Take derivative of flow equation
- $s_h$  = sensitivity of head
- $h$  = head
- $a$  = parameter

$$s_h = \partial h / \partial a$$



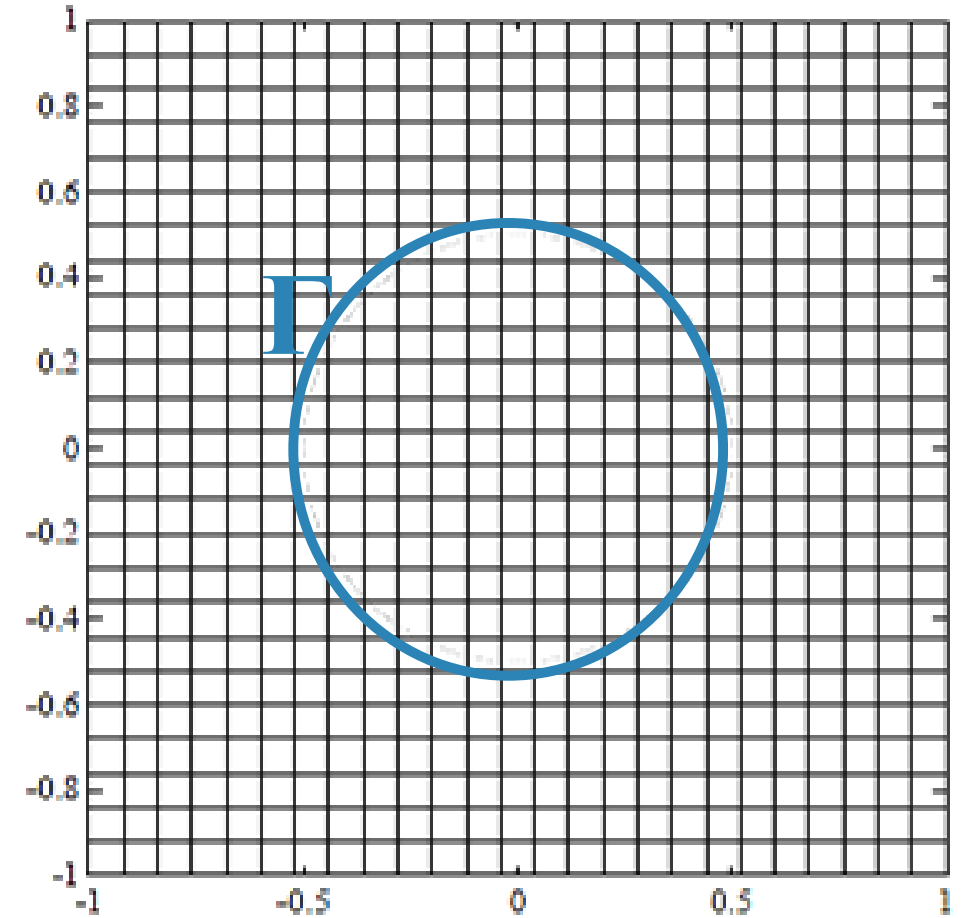
# How CSE Works

- Take full derivative of boundary conditions
- $u$  = boundary condition
- $a$  = parameter
- $\mathbf{x}$  = boundary location

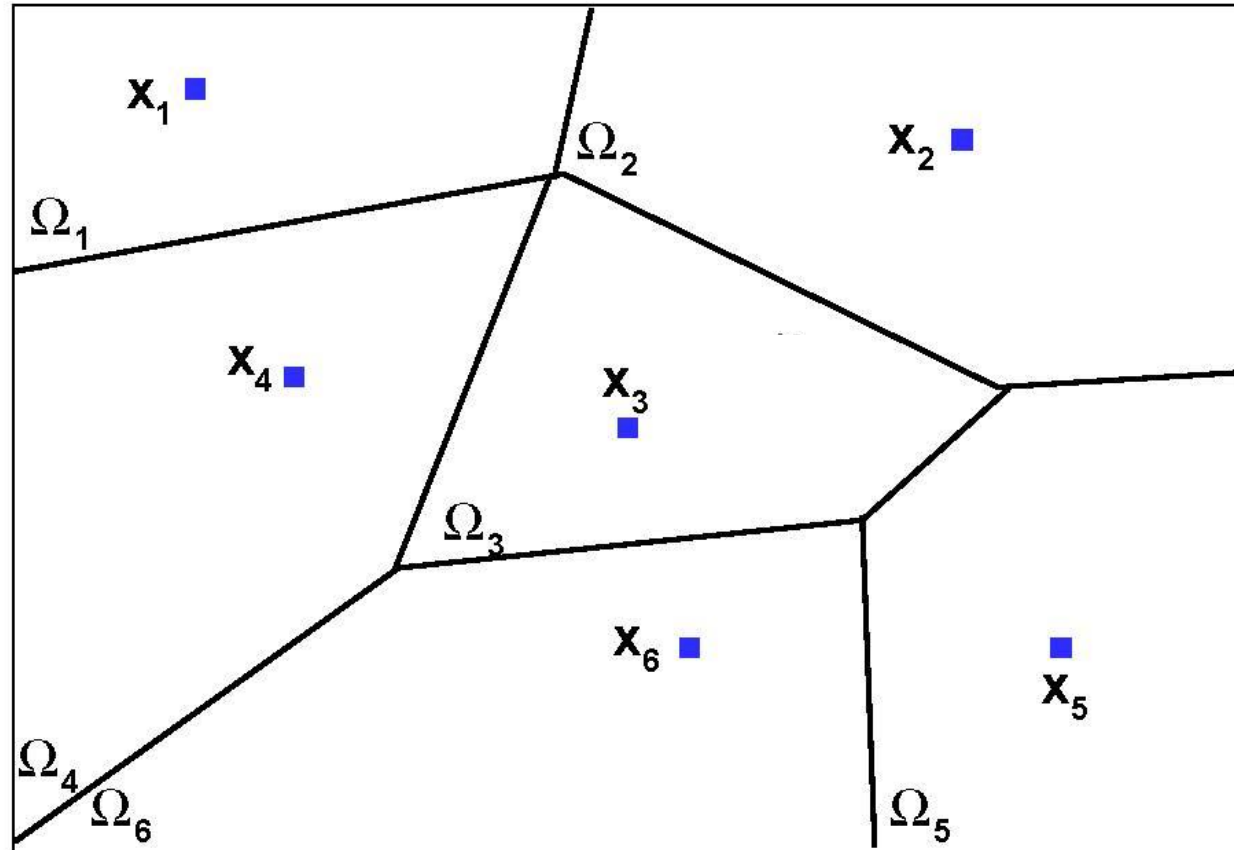
$$\frac{Du}{Da} = \frac{\partial u}{\partial a} + \nabla u \cdot \frac{\partial \mathbf{x}}{\partial a},$$

# Implementation

## Immersed Interface Method (Li, 1994)



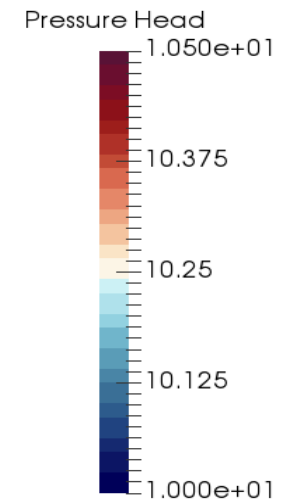
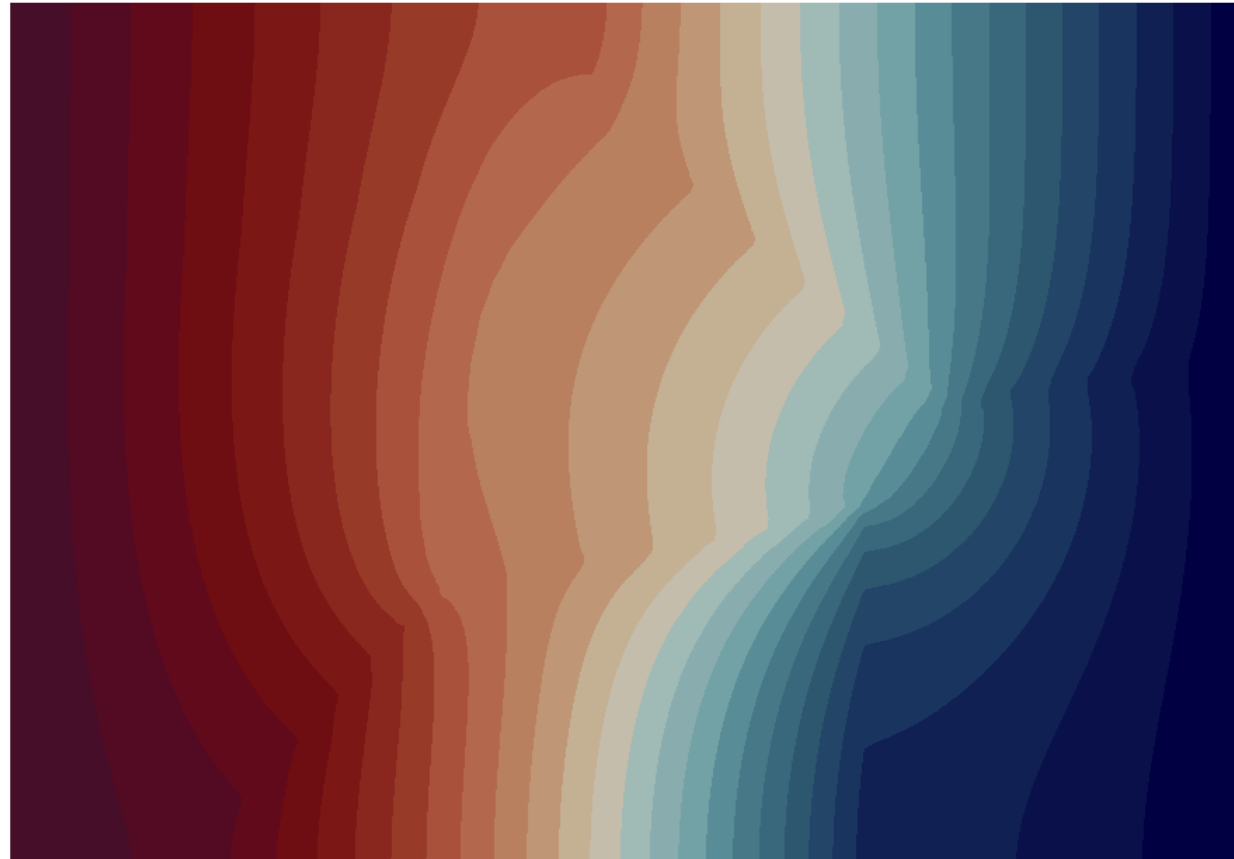
# Example



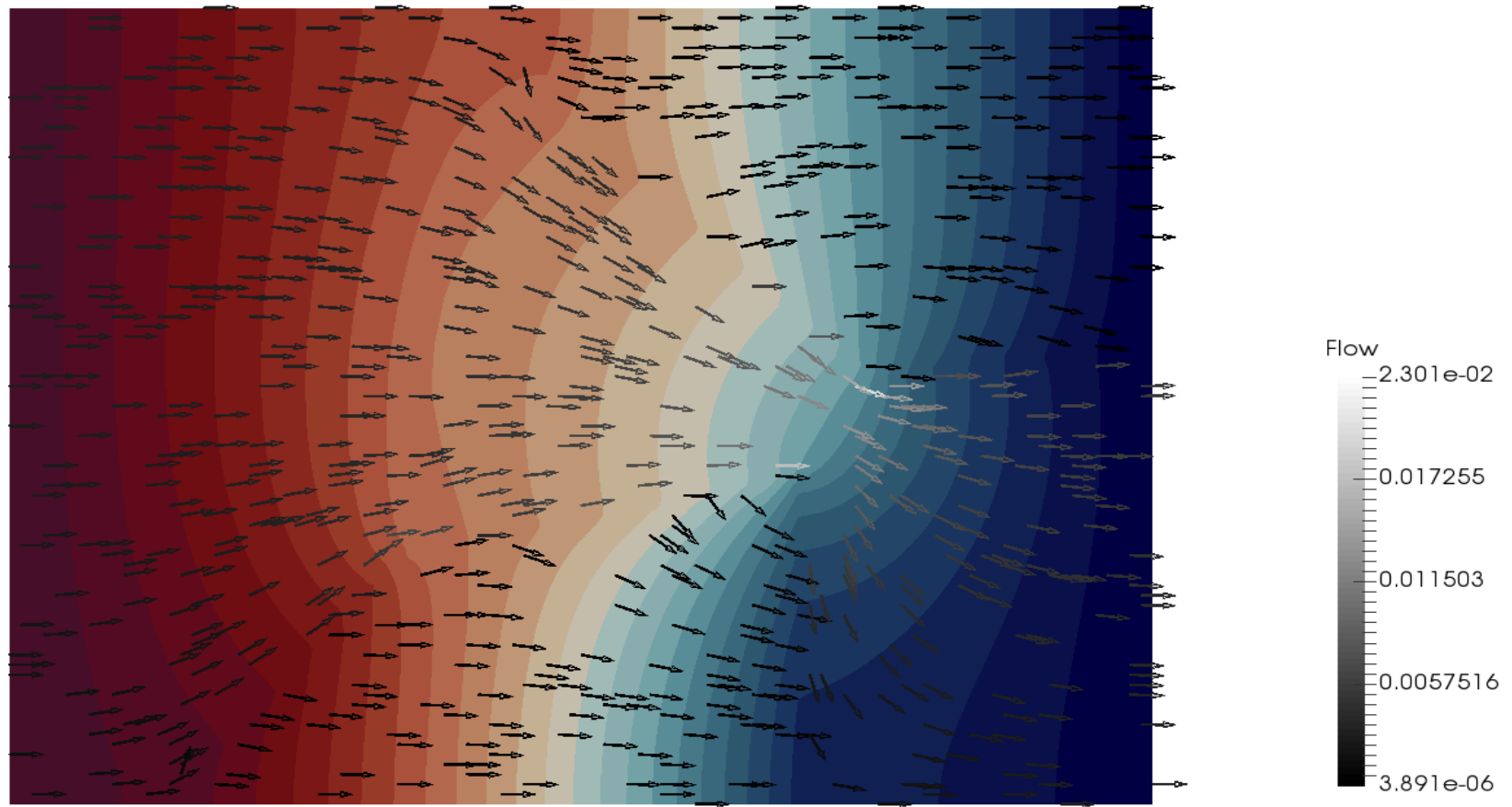
# Example



# Example



# Example



# Conclusions

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- Using the Continuous Sensitivity Equation can give an accurate, analytical solution for sensitivity
- CSE can be implemented using Immersed Interface Method

# Thank You!

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- My Mentor Lu Zhiming
- DOE Minority Serving Institution Partnership Program
- All the great EES-16 people I met